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Finite temperature properties of the Dirac operator with bag boundary conditions

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Abstract

We study the finite temperature free energy and fermion number for Dirac fields in a one-dimensional spatial segment, under local boundary conditions, compatible with the presence of a spectral asymmetry. We discuss in detail the contribution of this part of the spectrum to the determinant. We evaluate the finite temperature properties of the theory for arbitrary values of the chemical potential.

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1. Introduction

When the Euclidean Dirac operator is considered on even-dimensional compact manifolds with boundary, its domain can be determined through a family of local boundary conditions which define a self-adjoint boundary problem [1] (the particular case of two-dimensional manifolds was first studied in [2]). The whole family is characterized by a real parameter θ, which can be interpreted as an analytic continuation of the well-known θ parameter in gauge theories. These boundary conditions can be considered to be the natural counterpart in Euclidean space of the well-known chiral bag boundary conditions.

One salient characteristic of these local boundary conditions is the generation of an asymmetry in the spectrum of the Dirac operator. For the particular case of two-dimensional product manifolds, such asymmetry was shown, in [3], to be determined by the asymmetry of the boundary spectrum. For other recent work on chiral bag boundary conditions, see [4–6].

In a previous paper [7], we studied a theory of Dirac fields in one spatial dimension and evaluated its finite temperature properties for two particular values of the parameter θ, and for restricted ranges of the chemical potential μ, since our aim was to (partially) answer the question posed in [8], as to whether the fermion number is modified by temperature in low

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dimensional bags. Here, we generalize such results to other values of μ. Such generalization could be of interest in the study of boundary effects in low dimensional condensed matter systems and also in the treatment of open string theories with a non-trivial twist (analytic extension of μ) of the world sheet [9, 10].

In section 2, we determine the spectrum of the Euclidean Dirac operator at finite temperature for $\theta = 0$. With this spectrum at hand we perform, in section 3, the calculation of the partition function via zeta function regularization for different ranges of the chemical potential.

Section 4 is devoted to the evaluation of the free energy and the mean fermion number, both at finite and zero temperature.

2. Spectrum of the Euclidean Dirac operator

In order to study the effect of temperature, we consider a two-dimensional Euclidean space, with the metric (+, +). We take the Euclidean gamma matrices to be $\gamma_0 = \sigma_1$, $\gamma_1 = \sigma_2$. Thus, the Euclidean action is

$$S_E = \int \mathrm{d}^2x\, \bar{\Psi}(\mathrm{i}\not{\partial} - \not{A})\Psi. \tag{1}$$

The partition function is given by

$$\log Z = \log\det(\mathrm{i}\not{\partial} - \not{A})_{\mathrm{BC}}. \tag{2}$$

Here, BC stands for antiperiodic boundary conditions in the 'time' direction ($0 \leqslant x_0 \leqslant \beta$, with $\beta = \frac{1}{T}$) and, in the 'space' direction ($0 \leqslant x_1 \leqslant L$),

$$\tfrac{1}{2}(1+\gamma_0)\Psi\rfloor_0 = 0 \qquad \tfrac{1}{2}(1+\gamma_0)\Psi\rfloor_L = 0. \tag{3}$$

We will follow [11] in introducing the chemical potential as an imaginary $A_0 = -\mathrm{i}\mu$.

In order to evaluate the partition function in the zeta regularization approach, we first determine the eigenfunctions, and the corresponding eigenvalues, of the Dirac operator

$$(\mathrm{i}\not{\partial} + \mathrm{i}\gamma_0\mu)\Psi = \omega\Psi. \tag{4}$$

To satisfy antiperiodic boundary conditions in the x_0 direction, we expand

$$\Psi(x_0, x_1) = \sum_{\lambda} \mathrm{e}^{\mathrm{i}\lambda x_0}\psi(x_1), \tag{5}$$

with

$$\lambda_l = (2l+1)\frac{\pi}{\beta}, \qquad l = -\infty, \ldots, \infty. \tag{6}$$

After doing so, and writing $\psi(x_1) = \binom{\varphi(x_1)}{\chi(x_1)}$ we have, for each l,

$$(-\tilde{\lambda}_l + \partial_1)\chi = \omega\varphi \qquad (-\tilde{\lambda}_l - \partial_1)\varphi = \omega\chi, \tag{7}$$

where

$$\tilde{\lambda}_l = \lambda_l - \mathrm{i}\mu = (2l+1)\frac{\pi}{\beta} - \mathrm{i}\mu. \tag{8}$$

It is easy to see that, with the boundary condition in equation (3), no zero mode appears. For $\omega \neq 0$ one has, from (7),

$$\partial_1^2\varphi = -\kappa^2\varphi \qquad \chi = -\frac{1}{\omega}(\tilde{\lambda}_l + \partial_1)\varphi, \tag{9}$$

where $\kappa^2 = \omega^2 - \tilde{\lambda}_l^2$.

For $\kappa \neq 0$, one has for the eigenvalues

$$\omega_{n,l} = \pm\sqrt{\left(\frac{n\pi}{L}\right)^2 + \tilde{\lambda}_l^2}, \qquad \text{with} \quad n = 1, \ldots, \infty, \quad l = -\infty, \ldots, \infty. \tag{10}$$

This part of the spectrum is symmetric. In the case $\kappa = 0$ one has a set of x_1-independent eigenfunctions, corresponding to

$$\omega_l = \tilde{\lambda}_l. \tag{11}$$

It is to be noted that $\omega_l = -\tilde{\lambda}_l$ are not eigenvalues.

3. Partition function

In order to obtain the partition function, as defined in equation (2), we must consider the contributions to $\log \mathcal{Z}$ coming from both types of eigenvalues (equations (10) and (11)),

$$\Delta_1 = -\frac{\mathrm{d}}{\mathrm{d}s}\bigg]_{s=0} \zeta_1(s), \tag{12}$$

where

$$\zeta_1(s) = (1 + (-1)^{-s}) \sum_{\substack{n=1 \\ l=-\infty}}^{\infty} \left[\left(\frac{n\pi}{\alpha L}\right)^2 + \left((2l+1)\frac{\pi}{\alpha\beta} - \mathrm{i}\frac{\mu}{\alpha}\right)^2\right]^{-\frac{s}{2}}, \tag{13}$$

and

$$\Delta_2 = -\frac{\mathrm{d}}{\mathrm{d}s}\bigg]_{s=0} \zeta_2(s), \tag{14}$$

where

$$\zeta_2(s) = \sum_{l=-\infty}^{\infty} \left[(2l+1)\frac{\pi}{\alpha\beta} - \mathrm{i}\frac{\mu}{\alpha}\right]^{-s}. \tag{15}$$

As usual, α is a parameter with dimensions of mass, introduced to render the zeta function dimensionless.

The analytic extension of ζ_2 requires a careful selection of the cut in the ω-plane (for details, see [7]). The result is

$$\begin{aligned}
\zeta_2(s) &= \left(\frac{2\pi}{\alpha\beta}\right)^{-s} \left[\zeta_H\left(s, \frac{1}{2} - \frac{\mathrm{i}\mu\beta}{2\pi}\right) + \sum_{l=0}^{\infty}\left[-\left(l + \frac{1}{2}\right) - \mathrm{i}\frac{\mu\beta}{2\pi}\right]^{-s}\right] \\
&= \left(\frac{2\pi}{\beta\alpha}\right)^{-s} \left[\zeta_H\left(s, \frac{1}{2} - \frac{\mathrm{i}\mu\beta}{2\pi}\right) + \mathrm{e}^{\mathrm{i}\pi\,\mathrm{sign}(\mu)s}\zeta_H\left(s, \frac{1}{2} + \frac{\mathrm{i}\mu\beta}{2\pi}\right)\right],
\end{aligned} \tag{16}$$

where $\zeta_H(s, x)$ is the Hurwitz zeta function.

The analytic extension of ζ_1 leads to a separate consideration of different μ-ranges, determined by the energies of the zero-temperature problem [7]. We will perform the extension in two of these ranges. The generalization to other ranges will become evident from these two cases.

3.1. $|\mu| < \frac{\pi}{L}$

This is the range already studied by us. For details, see [7]. In this range, (13) can be written in terms of its Mellin transform as

$$\zeta_1(s) = \frac{(1+(-1)^{-s})}{\Gamma\left(\frac{s}{2}\right)} \int_0^\infty \mathrm{d}t\, t^{\frac{s}{2}-1} \sum_{\substack{n=1\\ l=-\infty}}^{\infty} \exp\left(-t\left[\left(\frac{n\pi}{\alpha L}\right)^2 + \left((2l+1)\frac{\pi}{\alpha\beta} - \mathrm{i}\frac{\mu}{\alpha}\right)^2\right]\right). \quad (17)$$

This can also be written as

$$\zeta_1(s) = \frac{(1+(-1)^{-s})}{(\sqrt{\pi})^s\Gamma\left(\frac{s}{2}\right)} \sum_{n=1}^{\infty} \int_0^\infty \mathrm{d}t\, t^{\frac{s}{2}-1} \exp\left(-t\pi\left[\left(\frac{n}{\alpha L}\right)^2 + \left(\frac{1}{\alpha\beta} - \frac{\mathrm{i}\mu}{\alpha\pi}\right)^2\right]\right)$$
$$\times\, \Theta_3\left(\frac{-2t}{\alpha\beta}\left(\frac{1}{\alpha\beta} - \frac{\mathrm{i}\mu}{\alpha\pi}\right), \frac{4t}{(\alpha\beta)^2}\right), \quad (18)$$

where we have used the definition of the Jacobi theta function $\Theta_3(z,x) = \sum_{l=-\infty}^{\infty} \mathrm{e}^{-\pi x l^2}\mathrm{e}^{2\pi z l}$.

To proceed, we use the inversion formula for the Jacobi function,

$\Theta_3(z,x) = \frac{1}{\sqrt{x}}\,\mathrm{e}^{(\frac{\pi z^2}{x})}\Theta_3\left(\frac{z}{\mathrm{i}x}, \frac{1}{x}\right)$, and perform the integration over t, thus getting

$$\zeta_1(s) = \frac{(1+(-1)^{-s})\beta}{2\alpha^{-s}(\sqrt{\pi})^s\Gamma(\frac{s}{2})}\left[\Gamma\left(\frac{s-1}{2}\right)\frac{\pi^{\frac{1-s}{2}}}{L^{1-s}}\zeta_R(s-1)\right.$$
$$\left. + 4\left(\frac{\beta L}{2}\right)^{\frac{s-1}{2}} \sum_{n,l=1}^{\infty} (-1)^l \left(\frac{l}{n}\right)^{\frac{s-1}{2}} \cosh(\mu\beta l) K_{\frac{s-1}{2}}\left(\frac{nl\pi\beta}{L}\right)\right], \quad (19)$$

where ζ_R is the Riemann zeta function.

From (19) and (16) both contributions to $\log\mathcal{Z}$ in this range of μ can be obtained. They are given by

$$\Delta_1 = -\frac{\beta\pi}{12L} + \sum_{n=1}^{\infty} \log\left(1 + \mathrm{e}^{-\frac{2n\pi\beta}{L}} + 2\cosh(\mu\beta)\,\mathrm{e}^{-\frac{n\pi\beta}{L}}\right) \quad (20)$$

and

$$\Delta_2 = -\left[\zeta_H'\left(0, \frac{1}{2} - \frac{\mathrm{i}\mu\beta}{2\pi}\right) + \zeta_H'\left(0, \frac{1}{2} + \frac{\mathrm{i}\mu\beta}{2\pi}\right) + \mathrm{i}\pi\,\mathrm{sign}(\mu)\zeta_H\left(0, \frac{1}{2} + \frac{\mathrm{i}\mu\beta}{2\pi}\right)\right]$$
$$= \log 2 + \log\cosh\left(\frac{\mu\beta}{2}\right) - \frac{|\mu|\beta}{2}. \quad (21)$$

Putting both pieces together, we finally have

$$\log\mathcal{Z} = -\frac{\beta\pi}{12L} + \sum_{n=1}^{\infty} \log\left(1 + \mathrm{e}^{-\frac{2n\pi\beta}{L}} + 2\cosh(\mu\beta)\,\mathrm{e}^{-\frac{n\pi\beta}{L}}\right) + \log 2 + \log\cosh\left(\frac{\mu\beta}{2}\right) - \frac{|\mu|\beta}{2}. \quad (22)$$

3.2. $\frac{\pi}{L} < |\mu| < \frac{2\pi}{L}$

Again, we have

$$\zeta_1(s) = \frac{(1+(-1)^{-s})}{\Gamma\left(\frac{s}{2}\right)} \int_0^\infty \mathrm{d}t\, t^{\frac{s}{2}-1} \sum_{\substack{n=1\\ l=-\infty}}^{\infty} \exp\left(-t\left[\left(\frac{n\pi}{\alpha L}\right)^2 + \left((2l+1)\frac{\pi}{\alpha\beta} - \mathrm{i}\frac{\mu}{\alpha}\right)^2\right]\right). \quad (23)$$

However, in this range of μ, the contribution to the zeta function due to $n = 1$ must be analytically extended in a different way. In fact, the expression cannot be written in terms of a unique Mellin transform, since its real part is not always positive (note, in connection with this that, for $n = 1$, equation (19) diverges). Instead, it can be written as a product of two Mellin transforms

$$\zeta_1^{n=1}(s) = \frac{(1+(-1)^{-s})}{\alpha^{-s}\left[\Gamma\left(\frac{s}{2}\right)\right]^2}\sum_{l=0}^{\infty}\int_0^{\infty} \mathrm{d}t\, t^{\frac{s}{2}-1}\exp\left(-\left[(2l+1)\frac{\pi}{\beta} - \mathrm{i}\mu + \mathrm{i}\frac{\pi}{L}\right]t\right)$$
$$\times \int_0^{\infty}\mathrm{d}z\, z^{\frac{s}{2}-1}\exp\left(-\left[(2l+1)\frac{\pi}{\beta} - \mathrm{i}\mu - \mathrm{i}\frac{\pi}{L}\right]z\right) + \mu \to -\mu \tag{24}$$

or, after changing variables according to $t' = t - z$; $z' = t + z$, performing the integral over t', and the sum over l

$$\zeta_1^{n=1}(s) = \frac{(1+(-1)^{-s})\sqrt{\pi}}{2\alpha^{-s}\Gamma\left(\frac{s}{2}\right)}\left(2\frac{\pi}{L}\right)^{\frac{1-s}{2}}\int_0^{\infty}\mathrm{d}z\, z^{\frac{s-1}{2}} J_{\frac{s-1}{2}}\left(\frac{\pi}{L}z\right)\frac{\mathrm{e}^{\mathrm{i}\mu z}}{\sinh\left(\frac{\pi z}{\beta}\right)} + \mu \to -\mu. \tag{25}$$

Now, the integral in this expression diverges at $z = 0$. In order to isolate such divergence, we add and subtract the first term in the series expansion of the Bessel function, thus getting the following two pieces:

$$\zeta_{1,(1)}^{n=1}(s) = \frac{(1+(-1)^{-s})\sqrt{\pi}s}{4\alpha^{-s}\Gamma\left(\frac{s}{2}+1\right)}\left(2\frac{\pi}{L}\right)^{\frac{1-s}{2}}\int_0^{\infty}\mathrm{d}z\, z^{\frac{s-1}{2}}\left[J_{\frac{s-1}{2}}\left(\frac{\pi}{L}z\right) - \frac{\left(\frac{\pi z}{2L}\right)^{\frac{s-1}{2}}}{\Gamma\left(\frac{s+1}{2}\right)}\right]$$
$$\times \frac{\mathrm{e}^{\mathrm{i}\mu z}}{\sinh\left(\frac{\pi z}{\beta}\right)} + \mu \to -\mu, \tag{26}$$

and

$$\zeta_{1,(2)}^{n=1}(s) = \frac{(1+(-1)^{-s})\sqrt{\pi}}{2^s\alpha^{-s}\Gamma\left(\frac{s}{2}\right)\Gamma\left(\frac{s+1}{2}\right)}\int_0^{\infty}\mathrm{d}z\, z^{s-1}\frac{\mathrm{e}^{\mathrm{i}\mu z}}{\sinh\left(\frac{\pi z}{\beta}\right)} + \mu \to -\mu. \tag{27}$$

The contribution of equation (26) to the partition function can easily be evaluated by noting that the factor multiplying s is finite at $s = 0$. Thus, one has

$$\Delta_{1,(1)}^{n=1} = -\int_0^{\infty}\mathrm{d}z\, z^{-1}\left[\cos\left(\frac{\pi}{L}z\right) - 1\right]\frac{\mathrm{e}^{\mathrm{i}\mu z}}{\sinh\left(\frac{\pi z}{\beta}\right)} + \mu \to -\mu, \tag{28}$$

where we have used that $J_{-\frac{1}{2}}\left(\frac{\pi}{L}z\right) = \sqrt{\frac{2}{\pi\frac{\pi}{L}z}}\cos\left(\frac{\pi}{L}z\right)$. Now, in the term with $\mu \to -\mu$, one can change $z \to -z$ to obtain

$$\Delta_{1,(1)}^{n=1} = -\int_{-\infty}^{\infty}\mathrm{d}z\, z^{-1}\left[\cos\left(\frac{\pi}{L}z\right) - 1\right]\frac{\mathrm{e}^{\mathrm{i}\mu z}}{\sinh\left(\frac{\pi z}{\beta}\right)}. \tag{29}$$

This last integral is easy to evaluate in the complex plane, by carefully taking into account the sign of μ, as well as the fact that $\frac{\pi}{L} < |\mu|$ in closing the integration path, to obtain

$$\Delta_{1,(1)}^{n=1} = -2\sum_{l=1}^{\infty}\left[\frac{(-1)^l}{l}\cosh\left(\frac{\pi}{L}\beta l\right)\mathrm{e}^{-|\mu|\beta l} + \frac{(-1)^{l+1}}{l}\,\mathrm{e}^{-|\mu|\beta l}\right] \tag{30}$$

or, after summing the series

$$\Delta_{1,(1)}^{n=1} = \left\{\log\left(1 + \mathrm{e}^{-2|\mu|\beta} + 2\cosh\left(\frac{\pi}{L}\beta\right)\,\mathrm{e}^{-|\mu|\beta}\right) + |\mu|\beta - 2\log\left(2\cosh\left(\frac{\mu\beta}{2}\right)\right)\right\}. \tag{31}$$

In order to get the contribution coming from (27), the integral can be evaluated for $\Re s > 1$, which gives

$$\zeta_{1,(2)}^{n=1}(s) = \frac{(1+(-1)^{-s})\Gamma(s)\sqrt{\pi}(\alpha\beta)^s}{(2\pi)^s 2^{s-1}\Gamma\left(\frac{s}{2}\right)\Gamma\left(\frac{s+1}{2}\right)} \times \left[\zeta_H\left(s, \frac{1}{2}\left(1-\frac{\mathrm{i}\mu\beta}{\pi}\right)\right) + \zeta_H\left(s, \frac{1}{2}\left(1+\frac{\mathrm{i}\mu\beta}{\pi}\right)\right)\right]. \tag{32}$$

Its contribution to the partition function can now be obtained by using that $\zeta_H\left(0, \frac{1}{2}\left(1-\frac{\mathrm{i}\mu\beta}{\pi}\right)\right) + \zeta_H\left(0, \frac{1}{2}\left(1+\frac{\mathrm{i}\mu\beta}{\pi}\right)\right) = 0$ and the well-known value of $-\frac{\mathrm{d}}{\mathrm{d}s}\rfloor_{s=0}\zeta_H(s,x)$ [12], to obtain

$$\Delta_{1,(2)}^{n=1} = 2\log\left(2\cosh\left(\frac{\mu\beta}{2}\right)\right). \tag{33}$$

Summing up the contributions in equations (21), (31) and (33), as well as the contribution coming from $n \geqslant 2$, evaluated as in the previous subsection, one gets for the partition function

$$\log Z = \left\{\log\left(2\cosh\left(\frac{\mu\beta}{2}\right)\right) + \frac{|\mu|\beta}{2} + \log\left(1+\mathrm{e}^{-2|\mu|\beta} + 2\cosh\left(\frac{\pi}{L}\beta\right)\mathrm{e}^{-|\mu|\beta}\right) + \beta\frac{\pi}{L}\left(-\frac{1}{12}-1\right) + \sum_{n=2}^{\infty}\log\left(1+\mathrm{e}^{-2n\frac{\pi}{L}\beta} + 2\cosh(\mu\beta)\,\mathrm{e}^{-n\frac{\pi}{L}\beta}\right)\right\}. \tag{34}$$

At first sight, this result looks different from the one corresponding to $|\mu| < \frac{\pi}{L}$ (equation (22)). However, it is easy to see that both expressions coincide, the only difference being that the zero-temperature limit is explicitly isolated from finite-temperature corrections. Similar calculations lead to the same conclusion for other ranges of μ. In those cases where μ coincides exactly with one energy level, the result can be shown to be the same, but series such as those in equations (19) and (30) are only conditionally convergent.

4. Free energy and fermion number

From the results in the previous section, the free energy can be readily obtained. It is given by

$$F = -\frac{1}{\beta}\log Z = \frac{\pi}{12L} - \frac{1}{\beta}\left[\sum_{n=1}^{\infty}\log\left(1+\mathrm{e}^{-\frac{2n\pi\beta}{L}} + 2\cosh(\mu\beta)\,\mathrm{e}^{-\frac{n\pi\beta}{L}}\right) + \log 2 + \log\cosh\left(\frac{\mu\beta}{2}\right) - \frac{|\mu|\beta}{2}\right]. \tag{35}$$

It is continuous, in particular, at $|\mu| = k\frac{\pi}{L}, k = 0, \ldots, \infty$. In the low-temperature limit one has

$$F\left(\frac{k\pi}{L} < |\mu| < \frac{(k+1)\pi}{L}\right) \to_{\beta\to\infty} \frac{\pi}{12L} + k(k+1)\frac{\pi}{2L} - k|\mu|. \tag{36}$$

The fermion number is obtained as

$$N = \frac{1}{\beta}\frac{\partial\log\mathcal{Z}}{\partial\mu}. \tag{37}$$

It is given by

$$N = \left\{\sum_{n=1}^{\infty}\left[\frac{\mathrm{e}^{-\frac{n\pi\beta}{L}+\mu\beta}}{1+\mathrm{e}^{-\frac{n\pi\beta}{L}+\mu\beta}} - \frac{\mathrm{e}^{-\frac{n\pi\beta}{L}-\mu\beta}}{1+\mathrm{e}^{-\frac{n\pi\beta}{L}-\mu\beta}}\right] + \frac{1}{2}\tanh\left(\frac{\mu\beta}{2}\right) - \frac{1}{2}\mathrm{sign}(\mu)\right\}. \tag{38}$$

Note that it is not defined for $\mu = 0$, where both lateral limits differ. This originates from the indetermination of the phase of the determinant (equation (16)). From a physical point of view, this reflects the fact that the sign of μ distinguishes particles from antiparticles.

Particularly interesting is the discontinuous behavior of N in the zero-temperature limit, where μ is to be interpreted as a Fermi energy. In such a limit, one has

$$N\left(\frac{k\pi}{L} < |\mu| < \frac{(k+1)\pi}{L}\right) \rightarrow_{\beta\to\infty} -k\,\mathrm{sign}(\mu), \tag{39}$$

which coincides with the derivative of equation (36), and is consistent with Fermi statistics. For μ equal to an energy level, both lateral limits differ, and $N\left(|\mu| = \frac{k\pi}{L}\right)$ is undefined.

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References

[1] Wipf A and Duerr S 1995 Gauge theories in a bag *Nucl. Phys.* B **443** 201
[2] Hrasko P and Balog J 1984 The fermion boundary condition and the theta angle in QED in two-dimensions *Nucl. Phys.* B **245** 118
[3] Beneventano C G, Santangelo E M and Wipf A 2002 Spectral asymmetry for bag boundary conditions *J. Phys. A: Math. Gen.* **35** 9343
[4] Beneventano C G, Gilkey P, Kirsten K and Santangelo E M 2003 Strong ellipticity and spectral properties of chiral bag boundary conditions *J. Phys. A: Math. Gen.* **36** 11533
[5] Esposito G and Kirsten K 2002 Chiral bag boundary conditions on the ball *Phys. Rev.* D **66** 085014
[6] Marachevsky V N and Vassilevich D V 2004 Chiral anomaly for local boundary conditions *Nucl. Phys.* B **677** 535
[7] Beneventano C G and Santangelo E M 2004 Finite temperature properties of the Dirac operator under local boundary conditions *J. Phys. A: Math. Gen.* **37** 9261
[8] Dunne G V and Rao K 2003 Finite temperature induced fermion number for quarks in a chiral field *Phys. Rev.* D **67** 045013
[9] Lugo A 2004 About the stability of a $D4$–$\bar{D}4$ system *Nucl. Phys.* B **701** 299
[10] Polchinski J 2003 *String Theory* vol 2 (Cambridge: Cambridge University Press)
[11] Actor A 1985 Chemical potentials in gauge theories *Phys. Lett.* B **157** 53
[12] Gradshteyn L S and Ryzhik L M 2000 *Table of Integrals, Series and Products* (New York: Academic)